

**Semester -I**  
**REAL ANALYSIS**

**Course No: M-511**

**Credit hours-4**

**Objective:**

After a first course in real analysis in undergraduate program, the ideas of uniform continuity, uniform convergence and approximation by polynomials are crucial in analysis. In addition to the Functions of bounded variation and their integrators, the student has to learn differentiating functions from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ . The techniques of integration of a function with respect to another function and the basic ideas of finding a Fourier series are also included.

**Expected Outcomes:**

After studying the course

1. The student will understand and solve problems of uniform continuity, uniform convergence and will also test whether a function is of bounded variation or not.
2. The student will learn and solve problems about partial derivatives, directional derivatives, Jacobians, Inverse and implicit theorems.
3. The student will be able to calculate Riemann Stieltjes Integrals.
4. The student will be able to find the Fourier series and apply it.

**UNIT-I**

Convergence of a sequence of real numbers, Continuous functions, Uniform continuity, Examples, pointwise and uniform convergence, tests of uniform convergence, Cauchy criterion, Weierstrass M Test, uniform convergence and continuity, uniform convergence and relation to integration and derivatives Weierstrass approximation theorem, power series, functions expressible as power series.

**UNIT-II**

Differentiation in  $\mathbb{R}^n$ , Partial derivatives, Directional derivatives, sufficient condition for differentiability, chain rule, Mean value theorem, Jacobians, Contraction mapping principle, inverse function theorem, implicit function theorem, rank theorem, differentiation of integrals, Taylor theorem in many variables.

**UNIT-III**

Function of bounded variation, examples, total variation, Function of bounded variation expressed as difference of increasing functions, rectifiable paths, Riemann Stieltjes Integrals, properties and techniques, sufficient condition for existence of the integral, Necessary condition for existence of the integral, Mean value theorem for Riemann Stieltjes integrals, Reduction to Riemann integrals.

**UNIT-IV**

Basic concepts of Fourier series, Fourier series of even and odd functions, half range series, Fourier series on other intervals, orthogonal systems of functions, Theorem on best approximation, properties of Fourier coefficients, Riesz Fisher theorem, Riemann Lebesgue lemma, Dirichlet integral, Integral representation for the partial sum of a Fourier series. Convergence of Fourier series

**The course is covered by**

1. W. Rudin-Principles of Mathematical analysis-Mc Graw Hill, 3rd Ed,
2. T Apostol-Mathematical Analysis- Pearson; 2nd edition, .

**Books for Reference:**

3. Terrence Tao- Analysis-I - Hindustan book agency.
4. Terrence Tao -Analysis- II- Hindustan book agency
5. S C Malik, Savita Arora –Mathematical Analysis- New Age International -5<sup>th</sup> edition

# COMPLEX ANALYSIS

Course No: M-512

Credit Hours-4

**Objective:** The course is a second course in complex analysis as every undergraduate student learns basic complex analysis as a core course. The objective is to make the student understand both the theory and problem components of analytic functions, conformal mappings, complex integration theory, product developments and normal families

**Expected Outcomes:**

After studying this course the student will be able to

CO1. understand analytic function as a mapping on the plane, Mobius transformation and conformal mappings.

CO2. prove Cauchy theorem on various domains and learn the use of Cauchy integral formula and other results.

CO3. Find singularities and Evaluate contour integral using method of residues.

CO4. Learn about product development, analytic continuation and normal families.

## Unit-I

Review of analytic functions and basic properties, stereographic projections, mappings of elementary functions and cross ratio, Bilinear transformations and its properties, conformal mapping.

## Unit-II

Complex integration and simple version of Cauchy's theorem: Curves, parameterization, line integrals, Cauchy theorems (rectangle, triangle, circular disk), Cauchy integral formula, Liouville's theorem, Morera's theorem, Cauchy inequality, fundamental theorem of algebra, uniqueness and identity theorems, maximum modulus theorems, Gauss- mean value theorem, Schwartz lemma. Poisson integral formula.

## Unit-III

Calculus of residue: Laurent series, Classifications of singularities, evaluation of real integral, argument principle, Rouché's theorem, Hurwitz's theorem, open mapping theorem.

## Unit-IV

Infinite product, Weierstrass product development, Mittag-Leffler's theorem, Analytic continuation, Schwarz reflection principle, Normal families.

Course is covered by:

1. S Ponnusamy and Herb Silverman: Complex variables with Applications: Birkhauser, (2006) (Indian Edition 2012)

(Chapter-2: 2.4; Chapter-3; Chapter-4, Chapter-5: 5.1, 5.2; Chapter-7; Chapter-8; Chapter-9; Chapter-10:10.2; Chapter-11: 11.2; Chapter-12; Chapter-13: 13.1)

**Books for References:**

1. L. V. Ahlfors - Complex Analysis, McGraw Hill, 3rd Ed., 2017.
2. R V Churchill, J W Brown and R F Verhey- Complex Variables and Applications, McGraw Hill, 9th Ed., 2013.
3. J. B. Conway - Functions of one Complex Variable, Springer; 2nd ed. 1978, 7th printing 1995 edition.
4. E. M. Stein and R. Shakarchi, Complex Analysis: Princeton University Press, New Jersey, (2003)

## ALGEBRA-I

Course No: M-513

Credit Hours -4

**Objective:** The objective of the course is to augment the core courses offered in under graduate level in group theory and linear algebra in a different perspective.

**Expected Outcomes:**

After studying this course the student will be able to

CO1. Solve problems of basic group theory, group actions, automorphisms and sylow theory.

CO2. Understand problems of product and semi direct product of groups and solvable groups

CO3. Find eigen value and eigen vectors and calculate various canonical forms.

CO4. Handle problems of unitary, self adjoint, normal operators and bilinear forms.

### UNIT-I

Series of groups: Composition series and the Holder program, Transposition and the alternating group, Group actions: Group actions and permutations Representations, Cayley's Theorem, The Class equation, Automorphisms, The Sylow Theorems, The simplicity of alternating group.

### UNIT-II

Direct and semi Direct products and abelian groups: Direct products, The Fundamental Theorem of finitely generated abelian groups, Recognizing Direct product, semidirect product, Further topics in Group Theory: p-groups, Nilpotent groups, Solvable groups, A word on free Groups

### UNIT-III

Review of vector space fundamentals, matrix representation of linear transformations, Eigenvalue and eigen vectors, Minimal polynomial, diagonalisation, triangulable operators, nilpotent form, Jordan canonical form, rational canonical form,

### UNIT-IV

Inner product spaces orthogonality. Adjoint of a linear transformation, unitary operators, Self adjoint and normal operators, bilinear forms, matrix of a bilinear form, classification of bilinear forms.

The course is covered by

1. [David S. Dummit](#), [Richard M. Foote](#), Abstract Algebra, 3rd Paperback, Wiley, 2011.
2. V Sahay and V Bist : Linear Algebra, Narosa publishing House, second edition Books for reference
3. I.N. Herstein: Topics in Algebra, John Wiley and Sons; 2nd edition
4. J. B. Fraleigh: A first Course in Algebra, Pearson, 7th Ed., 2013.
5. J. Gallian: Contemporary Abstract algebra, Brooks/Cole; 8th edition
6. Hoffman and Kunz: Linear Algebra, Prentice Hall
7. Rao and Bhimasankaran: Linear algebra, Hindustan publishing house

## TOPOLOGY

**Course No: M-514**

**Credit Hours-4**

**Objective:** This is an introductory course in Topology. The objective of this course is to have knowledge on topological spaces, Continuity, connectedness, compactness and separation axioms. Topology on Quotient spaces, Product spaces and metric spaces are also discussed.. The student will also learn on basic ideas of algebraic topology in homotopy, fundamental groups and covering spaces. However the thrust is on learning the point set topology.

### **Expected Outcomes:.**

After taking the course the student will be able

- CO1. To understand the concept of a topological space, basis, subbasis with various examples and to understand new topologies like product topology, quotient topology, metric topology etc .
- CO2. To solve problems involving continuous maps , homeomorphisms between two spaces , connectedness and compactness.
- CO3 To deal with Hausdorff, regular, normal ,separable, first and second countable spaces and Lindelöf spaces.
- CO4 To understand homotopy, fundamental groups, and covering spaces.

### **Unit-I**

[ Review (without question in exam) of set theory, countable sets, uncountable sets, finite and infinite sets.]

Cartesian product of a family of sets, Axiom of choice and its equivalents (without proof), principle of induction.

Topological spaces, examples, open sets, closed sets, basis and subbasis for a topology, closure and interior of sets, subspace topology, order topology, Continuous functions, homeomorphisms, Product topology, quotient topology.

### **Unit-II,**

Metric topology, standard topology, uniform topology, lower limit topology, Connectedness, Examples, Local connectedness, Path-connectedness, connected subsets of real line, compact Spaces, Examples, locally compact spaces, sequential compactness, limit point compactness, compact subsets of real line

### **Unit-III**

Countability axioms, First and second countable spaces, separable and Lindöf spaces, Separation axioms, Regular & completely regular space, normal spaces, Urysohn Lemma, Urysohn metrization theorem Tychonoff Theorem, compactness in metric spaces, compact open topology.

### **Unit-IV**

Homotopy, path homotopy, lifting and extension problems, covering projection, covering space, examples, Fundamental Group, fundamental Group of circle.

The course is covered by

1. Munkres J R - Topology, A First Course: Pearson; 2nd edition, 2000.

### **Books for reference**

2. J. Dugundji, Topology, Allyn and Bacon Inc., Boston, 1978
3. K. D. Joshi : Introduction to General Topology (Wiley Eastern Limited).

4. M A Armstrong. Basic Topology. Springer, 1983.
5. O Viro, O Ivanov, V Kharlamov, and N Netsvetaev. Elementary Topology, a problem Text book, American Mathematical society.

## **MATLAB**

**Course No: M-515**

**Credit Hours-2**

### **Objectives:**

MATLAB has become essential in many undergraduate courses where practical component and computation is there. The objective of this course is to train students in fundamentals of MATLAB tool. The student should understand MATLAB graphic feature and its application. This will promote new teaching model that will help to develop programming skill and techniques to solve problems.

### **Course Outcomes**

After learning this course a student can

1. use basic MATLAB tools
2. plot different graphs in two-dimensions and three-dimensions.
3. Use the inbuilt array structures for calculations of algebra of matrices and solve the system of equations through various numerical methods.
4. Use different control flows for the writing of the simple programs and explore various applications to Numerical analysis and differential equations

#### **Unit – I**

Basics of MATLAB, Input – Output, File types – Platform dependence – General commands. Interactive Computation: Matrices and Vectors – Matrix and Array operations – Creating and Using Inline functions – Using Built-in Functions and On- line Help – Saving and loading data – Plotting simple graphs.

#### **Unit – II**

Programming in MATLAB: Scripts and Functions – Script files – Functions files – Language specific features – Advanced Data objects. Applications – Linear Algebra – Matrices: Eigenvalues and Eigenvectors, Similarity Transformation and Diagonalization, Curve fitting and Interpolation – Data analysis and Statistics

#### **Unit – III**

Graphics: Basic 2-D Plots – Using subplot to Layout multiple graphs - 3 - D Plots – Handle Graphics – Saving and printing Graphs – Errors Application– Symbolic math: creating symbolic objects and expression, changing the form of an existing symbolic expression: collect, expand, factor, simplify, pretty command

#### **Unit – IV**

Applications to Numerical differentiation and integrations, roots of polynomials, finding maximum and minimum of a function, Ordinary differential equations, Analytic solution of ODE, boundary value problem, PDE: -pdepe command Introduction to working with modules in MATLAB,

The course is covered by

1. RUDRA PRATAP, Getting Started with MATLAB – A Quick Introduction for Scientists and Engineers, Oxford University Press, 2003.

BOOK for reference

2. Amos Gilot: MATLAB an Introduction with applications Wiley 4<sup>th</sup> Edition 2011
3. Yang, W. Y., Cao, W., Chung, T. and Morris, J. Applied Numerical Methods using MATLAB. John Wiley Interscience, 2005

## PROGRAMMING LAB-I (MATLAB)

Course No: M-516

Credit-2

### Objective:

These practicals add to their undergraduate training of writing MATLAB CODES for various Mathematical problems

### Expected Outcome:

1. To learn to write codes using basics of MATLAB
2. To write code for problems from calculus and series sums.
3. To Write MATLAB codes for problems linear Algebra
4. To write MATLAB code for finding roots of equations, for problems in Numerical analysis.

### List

#### GROUP A

1. Write MATLAB code to find a root of the equation  $x^3 - 5x + 1 = 0$  by using Bisection method.
2. Write MATLAB code to find the solution of a nonlinear equation  $\tan(\pi - x) - x = 0$  by using Bisection method.
3. Using Bisection method find the roots of the following equation taking up to 50 iterations.  $X^2 + 2x - 2 = 0$ .
4. Write MATLAB code to find a root of the equation  $x^3 - 5x + 1 = 0$  by using Secant method.
5. Write MATLAB code to find a root of equation  $\sin x = e^x - 5$  by using Newton-Raphson method.
6. Write MATLAB code to find a root of the equation  $\cos x - xe^x = 0$  by using Newton Raphson method.
7. Write MATLAB code to find a root of the equation  $x^3 - 5x + 1 = 0$  by using Regula-Falsi Method.
8. Write MATLAB code to find a root of the equation  $\cos x - xe^x = 0$  by using Regula-Falsi Method.
9. Write MATLAB code to find the approximate value of  $\int_0^{\frac{\pi}{2}} \sin x dx$  by using the trapezoidal rule. Also compute the error.
10. Write MATLAB code to find the approximate value of  $\int_0^1 e^{-x} dx$  by using the trapezoidal rule. Also compute the error.
11. Write MATLAB code to find the approximate value of  $\int_0^1 \frac{1}{1+x} dx$  by using Simpson's 1/3rd Rule.
12. Write MATLAB code to find the approximate value of  $\int_0^1 \frac{1}{1+x} dx$  by using Simpson's 3/8 Rule.
13. Write MATLAB code to find the solution of the system of equations:  $4x_1 + x_2 + x_3 = 2$ ;  $x_1 + 5x_2 + 2x_3 = -6$ ;  $x_1 + 2x_2 + 3x_3 = -4$  by using Gauss-Jacobi iteration method with  $[0.5, -0.5, -0.5]^T$ .
14. Write MATLAB code to find the solution of the system of equations  $4x_1 + x_2 + x_3 = 2$ ;  $x_1 + 5x_2 + 2x_3 = -6$ ;  $x_1 + 2x_2 + 3x_3 = -4$  by using Gauss-Seidal iteration method.
15. Write MATLAB code to find the solution of the system of equations  $x_1 - 2x_2 + x_3 = 0$ ;  $2x_1 + x_2 - 3x_3 = 5$ ;  $4x_1 - 7x_2 + x_3 = -1$  by using Gauss elimination method.
16. Write MATLAB code to find the value of a function f at 2.2 using Lagrange interpolation

method, where  $f(0) = 1$ ,  $f(1) = 3$  and  $f(3) = 55$ .

17. Write MATLAB code to find the value of a function  $f$  at 13 using Lagrange interpolation method, where  $f(5) = 12$ ,  $f(6) = 13$ ,  $f(9) = 14$  and  $f(11) = 16$ .

#### GROUP B

1. Write MATLAB code to plot  $y = \sin(1/x)$  and  $y = \sin x + \cos x$  on a single figure window.
2. Write a MATLAB program to plot  $y = \cos(1/x)$  and  $y = \sin 2x - \cos(x/2)$  on a single figure window.
3. Write a MATLAB program to plot  $y = \sin(1/x)$  and  $y = x^2 + \exp x$  on a single figure window.
4. Write MATLAB code to Sketch the parametric curve Trochoid, cycloid and epicycloid.
5. Write MATLAB code to Sketch Ellipsoid, Hyperboloid and sphere of radius 2.
6. Write MATLAB code to the parametric curve Trochoid, cycloid and epicycloid.
7. Write MATLAB code to Sketch Ellipsoid, Hyperboloid and sphere of radius 5.3.
8. Write a MATLAB code to plot surface of revolution around x-axis.
9. Write a MATLAB code to plot  $\sin(4x-1)$ ,  $1/(ax+b)$  and  $|-7x + 78|$ .
10. Write a MATLAB code to plot  $\sin(ax + b)$ ,  $1/(ax+b)$  and  $|ax + b|$ .
11. Write a MATLAB code to plot the surface  $z = \frac{xy}{x^2+y^2}$  for  $-1 \leq x \leq 3$  and  $1 \leq y \leq 4$ .

12. Write MATLAB code to plot a function  $f(x,y) = \frac{xy}{x^2+y^2}$  .
13. Write MATLAB code to plot the surface  $z = \frac{xy(x^2-y^2)}{x^2+y^2}$  for  $-3 \leq x \leq 3$  and  $-3 \leq y \leq 3$ .
14. Write MATLAB code to plot the surface  $z = \frac{y^2(x^2-y^2)}{x^2+y^2}$  for  $-2 \leq x \leq 2$  and  $-2 \leq y \leq 2$ .
15. Write a MATLAB code to trace Parabola and Ellipse.
16. Write a program to find one-norm and two-norm of a given  $3 \times 3$  Matrix.
17. Write a program to find one-norm and two-norm of a given  $5 \times 5$  Matrix.
18. Write MATLAB code to plot the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  where  $a = b = 5$  and  $c = 3$ .

Students are required to maintain practical records